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AN EFFECTIVE AND SIMPLE ALGORITHM TO SOLVE THE DISCRETE MULTI-PRODUCT ECONOMIC PRODUCTION QUANTITY MODEL

Abstract. *This study proposes a practical heuristic algorithm to solve a complex and hard nonlinear integer programming (NLIP) formulation developed for a lot-sizing problem in multi-product economic production quantity (EPQ) environments containing a company and a supplier with delivery order constraints. In this paper, the previously published model is being modified with fewer constraints and decision variables, in order to find a better solution with less computational time using the proposed heuristic. We also show that the heuristic algorithm can find the optimal solution of a single-product inventory control problem without constraints.*

Keywords: *EPQ, multiple products, multiple deliveries, lot-sizing, NLIP.*

JEL Classification: 90B05

1. Introduction

In the beginning of the twentieth century, two of the primary mathematical inventory models called the economic order quantity (EOQ) and economic production quantity (EPQ) were presented by Harris in 1913 and Taft in 1918 (Nobil and Taleizadeh, 2016). These models are the simplest and the most fundamental in inventory models. After that, researchers extended them to suit real-world conditions such as the ones in multi-item inventory problems, manufacturing systems with defective items, discrete delivery orders, etc. (see for instance,Pasandideh et al., 2015;Nobil et al. 2016; Huang et al., 2017; Nobil et al., 2017; Karmakar et al., 2017).

The current study revisits the multi-product EPQ model with constraints on warehouse space and number of shipments proposed by Pasandideh and Niaki (2010). This problem considers a company that works with a supplier. The supplier provides all items and sends them to the company using some pallets. They formulated the problem into a nonlinear integer programming framework and proposed a genetic algorithm to find a near-optimum solution of the problem. In addition, the single product EPQ model with discrete delivery orders proposed by Pasandideh and Niaki (2010) is revisited as well. The model developed in their paper was an extension of the model in Pasandideh and Niaki (2008) by considering a single-product inventory system without constraints. Therefore, they presented an exact method to solve the problem. Later, Widyadana and Wee (2009) extended Pasandideh and Niaki's (2010) model to be used in an integrated single-vendor single-buyer inventory problem in just-in-time environments. Therafter,Cárdenas-Barrón et al. (2014) proposed a simple and better heuristic algorithm to solve the mixed-integer nonlinear programming (MINLP) formulation developed by Widyadana andWee (2009).

The above NLIP and MINLP problems that involve nonlinear constraints are hard to be solved using an exact method. That is why Pasandideh and Niaki (2008) employed a meta-heuristic algorithm called GA to solve their complicated optimization problem consisting of a nonlinear objective function and $n + 1$ non-linear constraints. Although GAhas a wide applicability to solvecomplexoptimization problemswith non-linear objective functions, they are expensive to be used in some cases (Cárdenas-Barrón et al., 2012).

In this research, it is first observed that the mathematical formulation proposed by Pasandideh and Niaki (2008) has some shortcomings. Then, a modification is proposed to remove the shortcomings. As the modified model will be shown to have fewer constraints and decision variables, the computational time of solving the problem using a novel heuristic becomes less compared to the one provided in the original formulation. In addition, more simple calculations and better solutions are obtained. Moreover, the proposed heuristic can find the optimal solution of the problem in Pasandideh and Niaki (2010) if it is used for a single-product production-inventory system without constraints. It will be also shown that the heuristic has better computational speed and less calculation compared to the one in Pasandideh and Niaki (2010).

2. Discussion

The same assumptions and notations used in Pasandideh and Niaki (2008) are adopted in this paper. It is assumed that a company works with a supplier. The supplier provides all items and sends them to the company using some pallets. The warehouse space of the company for all items is limited. Besides, the company determines the capacity of each pallet and the number of shipments for each item. The total inventory cost consists of total setup costs, total transportation costs, total providence costs and total holding costs for all items. The notation used in their model for item $i; i = 1,2, \dots, n$, are as follows:

n	Number of products
Q_i	Order quantity of item i
D_i	Demand rate of item i
P_i	Production rate of item i i
t_i	Time between two sequential shipments of each pallet for item i
l_i	Minimum number of shipments in each cycle for item i
u_i	Maximum number of shipments in each cycle for item i
f	Available warehouse space for all items
f_i	Space occupied by each unit of item i i
m_i	Number of shipments in each cycle for item i(decision variables)
k_i	Capacity of a pallet for item i(decision variables)
b_i	Transportation cost of a shipment for item i
h_i	Holding cost per unit per unit time of item i
A_i	Setup cost of each cycle for item i
c_i	Providence cost per unit of item i
TC	Total inventory cost for all items per year

Based on the notation and problem definition described above, Pasandideh and Niaki (2008) proposed the following formulation for the multi-product manufacturing-inventory problem at hand with discrete delivery orders.

$$Min\ TC = \sum_{i=1}^{n} \left(c_i D_i + \frac{b_i D_i}{k_i} + \frac{A_i D_i}{Q_i} + \frac{h_i}{2}\left(Q_i - (Q_i - k_i)\frac{D_i}{P_i}\right)\right) \tag{1}$$

$$s.t. \qquad \sum_{i=1}^{n} (f_i Q_i) \le f, \tag{2}$$

$$l_i \le m_i \le u_i; \qquad i = 1,2, \dots, n, \tag{3}$$

$$Q_i = m_i k_i; \qquad i = 1,2, \dots, n, \tag{4}$$

$$Q_i, m_i, k_i > 0; \qquad i = 1,2, \dots, n, \qquad integer \tag{5}$$

In the proposed model, the objective function in Eq. (1) is to minimize the total inventory cost of all items consisting of the total transportation cost, total setup cost, total Providence cost and total holding cost. Constraint (2) is the warehouse space limitation. Constraint (3) is the minimum and the maximum boundaries for the number of shipped pallets for each item. Constraint (4) defines the order quantity of each item in a cycle. Constraint (5) states that Q_i, m_i and k_i for all items are discrete and positive variables.

Technically, Constraint (4) was brought by to show that the lot size of each item (Q_i) depends on the number of shipment (m_i) and the capacity of a pallet (k_i) for that item; each a positive integer number as shown in Constraint (5). This limitation makes Q_i also an integer, as it is obtained by the multiplication of two positive integers. A simple way to reduce the number of decision variables and constraints significantly is to omit Constraint (4) and substitute $Q_i$by $m_i k_i$. This results in reducing the number of constraints and the decision variables, each by n. As such, the problem becomes more simple and faster to solve. Thus, the following simple modification is proposed for Pasandideh and Niaki's (2008) formulation.

$$Min\ TC = \sum_{i=1}^{n}\left(c_i D_i + \frac{b_i D_i}{k_i} + \frac{A_i D_i}{m_i k_i} + \frac{h_i D_i}{2P_i} k_i + \frac{h_i}{2}\left(1 - \frac{D_i}{P_i}\right) m_i k_i\right) \tag{6}$$

$$s.t.: \quad \sum_{i=1}^{n} f_i m_i k_i \le f, \tag{7}$$

$$l_i \le m_i \le u_i; \quad i = 1,2,\dots,n, \tag{8}$$

$$m_i, k_i > 0;\ i = 1,2,\dots,n, \quad integer \tag{9}$$

Moreover, the NLIP problem shown in (6)-(9) makes the problem studied in Pasandideh and Niaki (2010) simpler, if the manufacturing-inventory system includes a single product without Constraints (7) and (8).

In what follows, a heuristic algorithm is proposed to solve the problem modeled by the modified NLIP problem in (6)-(9).

3. Solution algorithm

In order to develop the heuristic, the objective function in (6) can be first rewritten as:

$$TC = \sum_{i=1}^{n}(Z_i + W_i + c_i D_i) \tag{10}$$

where $c_i D_i$ is fixed and we have:

$$Z_i = \frac{b_i D_i}{k_i} + \frac{h_i D_i}{2P_i} k_i \tag{11}$$

And

$$W_i = \frac{A_i D_i}{m_i k_i} + \frac{h_i}{2}\left(1 - \frac{D_i}{P_i}\right) m_i k_i. \tag{12}$$

It is noticeable that both Z_i and W_i have a similar expression of the form $v_1 R + v_2/R$; where both v_1 and v_2 are positive and R is a positive integer number ($R \ge 1, integer$). In addition, the optimal discrete value of R can always be obtained by minimizing $v_1 R + v_2/R$ as shown by García-Laguna et al. (2010), where both v_1 and v_2 are positive. In other words, the optimal value of R can be obtained using

$$R = \left\lceil -0.5 + \sqrt{0.25 + \frac{v_2}{v_1}} \right\rceil \;\; or \;\; R = \left\lfloor 0.5 + \sqrt{0.25 + \frac{v_2}{v_1}} \right\rfloor, \tag{13}$$

where $\lceil R \rceil$ is the samllest integer greater than or equal to R, and $\lfloor R \rfloor$ is the largest integer less than or equal to R.

For the problem at hand, if $\left(-0.5 + \sqrt{0.25 + \frac{v_2}{v_1}}\right)$ is not integer, the problem has a unique solution for R as $R^* = \left\lceil -0.5 + \sqrt{0.25 + \frac{v_2}{v_1}} \right\rceil$. Otherwise, both $R^* = \left(-0.5 + \sqrt{0.25 + \frac{v_2}{v_1}}\right)$ and $R^* = \left(\left\lceil -0.5 + \sqrt{0.25 + \frac{v_2}{v_1}} \right\rceil + 1\right)$ are optimal integer solutions. Thus, based on Eq. (13), the solution for each discrete variable (k_i) with respect to Eq. (11) is as follows:

$$k_i = \left\lceil -0.5 + \sqrt{0.25 + \frac{2b_i P_i}{h_i}} \right\rceil \;\; or \;\; k_i = \left\lfloor 0.5 + \sqrt{0.25 + \frac{2b_i P_i}{h_i}} \right\rfloor. \tag{14}$$

Based on Eq. (13) and given the discrete value of each k_i, the solution for each discrete variable (m_i) is hence determined as:

$$m_i = \left\lceil -0.5 + \sqrt{0.25 + \frac{2A_i D_i}{h_i k_i^2 \left(1 - \frac{D_i}{P_i}\right)}} \right\rceil \;\; or \;\; m_i = \left\lfloor 0.5 + \sqrt{0.25 + \frac{2A_i D_i}{h_i k_i^2 \left(1 - \frac{D_i}{P_i}\right)}} \right\rfloor. \tag{15}$$

Finally, TC can be written as:

$$TC = \sum_{i=1}^{n} (Z_i + c_i D_i), \tag{16}$$

where

$$Z_i = \left(b_i D_i + \frac{A_i D_i}{m_i}\right)\frac{1}{k_i} + \left(\frac{h_i D_i}{2P_i} + \frac{h_i}{2}\left(1 - \frac{D_i}{P_i}\right) m_i\right) k_i. \tag{17}$$

Executing the prior results, k_i can be computed as

$$k_i = \left\lceil -0.5 + \sqrt{0.25 + \frac{2D_i\left(b_i + \frac{A_i}{m_i}\right)}{\left(\frac{h_i D_i}{P_i} + h_i\left(1 - \frac{D_i}{P_i}\right) m_i\right)}} \right\rceil \text{ or}$$
$$k_i = \left\lfloor 0.5 + \sqrt{0.25 + \frac{2D_i\left(b_i + \frac{A_i}{m_i}\right)}{\left(\frac{h_i D_i}{P_i} + h_i\left(1 - \frac{D_i}{P_i}\right) m_i\right)}} \right\rfloor. \tag{18}$$

Based on the above derivations, the heuristic algorithm proposed by Cárdenas-Barrón et al. (2012)can be tailor made to solve the modified NLIP problem. The steps involved in this algorithm follows:

***Step*1**. Compute the initial discrete value of each k_i using Eq. (14). Then, obtain the discrete value of each m_i by Eq. (15). Go to *Step* 2.

***Step*2**. If $l_i \leq m_i \leq u_i$ for each item, then set $m_i = m_i$. Elseif, $m_i < l_i$ then set$m_i = l_i$. Othewise, if $m_i > u_i$ then set $m_i = l_i$. Use Eq. (18) to compute the final discrete value of k_i. If the solution satifies the warehouse space constraint in (7), then go to *Step* 7. Otherwise, go to *Step* 3.

***Step*3**. Solve the optimization problem subject to the warehouse space constraint. Calculate the discrete value of each m_i using Eq. (24), where the Lagrange multiplier θ is determined by solving Eq. (25). (see the explanation given at the end of the steps.) Go to *Step* 4.

***Step*4**. If $l_i \leq m_i \leq u_i$ for each item, then set $m_i = m_i$. Elseif, $m_i < l_i$ then set$m_i = l_i$. Othewise, if $m_i > u_i$ then set $m_i = l_i$. Use Eq. (30) to compute the discrete value of k_i where θ is determined by solving Eq. (31). (see the explanation given at the end of the steps.) Go to *Step* 5.

***Step*5**. If the solution satisfies the warehouse space constraint, then go to *Step* 7. Otherwise, go to *Step* 6.

***Step*6**. If the solution does not satisfy the warehouse space constraint then:

set warehouse constraint to $\mathrm{f_{new}} \leftarrow \frac{\mathrm{f}}{2}$

else

if $|\mathrm{f_{new}} - \mathrm{f}| < \varepsilon$

go to *Step* 6,

else

set $\mathrm{f_{new}} = \frac{(\mathrm{f_{new}}+\mathrm{f})}{2}$

set $\mathrm{f} = \mathrm{f_{new}}$ and go to *Step* 1.

***Step*7**. Based ona solution, calculate the total inventory cost using Eq. (15) and show the solution.

In *Step* 3, the constraint optimization problem is solved using the Lagrange method with the multiplier θ as:

$$Min\ \ TC = \sum_{i=1}^{n}\left(c_iD_i + \frac{b_iD_i}{k_i} + \frac{A_iD_i}{m_ik_i} + \frac{h_iD_i}{2P_i}k_i + \frac{h_i}{2}\left(1-\frac{D_i}{P_i}\right)m_ik_i\right) + \theta\left(\sum_{i=1}^{n} f_im_ik_i - f\right) \tag{19}$$

In this problem, TC can be written as:

$$TC = \sum_{i=1}^{n} (Z_i + W_i + c_i D_i) + X \tag{20}$$

where

$$Z_i = \frac{b_i D_i}{k_i} + \frac{h_i D_i}{2P_i} k_i \tag{21}$$

$$W_i = \frac{A_i D_i}{m_i k_i} + \left(\frac{h_i}{2}\left(1 - \frac{D_i}{P_i}\right) + \theta f_i\right) m_i k_i \tag{22}$$

$$X = -\theta f \tag{23}$$

Having m_i as

$$\begin{aligned} m_i &= \left\lfloor -0.5 + \sqrt{0.25 + \frac{2A_i D_i}{k_i^2 \left[h_i \left(1 - \frac{D_i}{P_i}\right) + 2\theta f_i\right]}} \right\rceil \quad or \\ m_i &= \left\lfloor 0.5 + \sqrt{0.25 + \frac{2A_i D_i}{k_i^2 \left[h_i \left(1 - \frac{D_i}{P_i}\right) + 2\theta f_i\right]}} \right\rfloor \end{aligned} \tag{24}$$

where k_i is obtained by Eq. (14), the value of θ can be computed by solving the following equation:

$$\sum_{i=1}^{n} f_i \left\{ \left\lceil -0.5 + \sqrt{0.25 + \frac{2A_i D_i}{k_i^2 \begin{bmatrix} h_i \left(1 - \frac{D_i}{P_i}\right) \\ +2\theta f_i \end{bmatrix}}} \right\rceil \right\} \left\lceil -0.5 + \sqrt{0.25 + \frac{2b_i P_i}{h_i}} \right\rceil - f = 0 \tag{25}$$

Given the solution of each m_i from *Step* 3, the optimization problem is solved using the Lagrange method in *Step* 4 as:

$$\begin{aligned} Min\ \ TC = \sum_{i=1}^{n} \left(c_i D_i + \frac{b_i D_i}{k_i} + \frac{A_i D_i}{m_i k_i} + \frac{h_i D_i}{2P_i} k_i + \frac{h_i}{2}\left(1 - \frac{D_i}{P_i}\right) m_i k_i \right) \\ + \theta \left(\sum_{i=1}^{n} f_i m_i k_i - f \right) \end{aligned} \tag{26}$$

Once again, TC can be written as:

$$TC = \sum_{i=1}^{n} (Z_i + c_i D_i) + X \tag{27}$$

where

$$Z_i = \left(b_i D_i + \frac{A_i D_i}{m_i}\right)\frac{1}{k_i} + \left(\frac{h_i D_i}{2P_i} + \frac{h_i}{2}\left(1 - \frac{D_i}{P_i}\right)m_i + \theta f_i m_i\right)k_i \tag{28}$$

$$X = -\theta f \tag{29}$$

Executing the prior results, k_i can be calculated by

$$k_i = \left\lceil -0.5 + \sqrt{0.25 + \frac{2D_i\left(b_i + \frac{A_i}{m_i}\right)}{\left(\frac{h_i D_i}{P_i} + h_i\left(1 - \frac{D_i}{P_i}\right)m_i + 2\theta f_i m_i\right)}}\right\rceil \quad or$$
$$k_i = \left\lfloor 0.5 + \sqrt{0.25 + \frac{2D_i\left(b_i + \frac{A_i}{m_i}\right)}{\left(\frac{h_i D_i}{P_i} + h_i\left(1 - \frac{D_i}{P_i}\right)m_i + 2\theta f_i m_i\right)}}\right\rfloor \tag{30}$$

where m_i is obtained by Eq. (24) and the value of θ can be computed by solving the following equation:

$$\sum_{i=1}^{n} f_i m_i \left\{\left\lceil -0.5 + \sqrt{0.25 + \frac{2D_i\left(b_i + \frac{A_i}{m_i}\right)}{\left(\frac{h_i D_i}{P_i} + h_i\left(1 - \frac{D_i}{P_i}\right)m_i + 2\theta f_i m_i\right)}}\right\rceil\right\} - f = 0 \tag{31}$$

Note that if the above heuristic algorithm is used to solve a single-product production-inventory problem without constraints, the optimal solution obtained for the formulation proposed by Pasandideh and Niaki (2010) is obtained as follows:

***Step*1**. Calculate the initial discrete value of k using Eq. (14) and go to *Step* 2.

***Step*2**. Given the discrete solution of k, compute the optimal discrete value of m^* by Eq. (15) and go to *Step* 3.

. ***Step*3**. Given the optimal solution of m^*, compute the final discrete value of k^* using Eq. (18).

A numerical example is solved in the following section to demonstrate the application of the proposed methodology.

4. Numerical example

The numerical example used in Pasandideh and Niaki (2008) is borrowed in this paper for illustration. They solved this example by a genetic algorithm (GA), where no final answers were given for the decision variables in their study. The parameters of this example are shown in Table 1. Moreover, the l_i and u_i values are assumed the same for all items as 5 and 35, respectively. The available warehouse space is set to 7900.

The steps taken to solve the problem using the proposed heuristic algorithm are described as follows.

Table 1. Parameters of the example in Pasandideh and Niaki (2008) stated

Item	P_i	D_i	A_i	h_i	b_i	c_i	f_i
1	66	21	30	4	6	19	5
2	57	18	88	9	2	23	8
3	71	27	71	7	9	37	4
4	29	16	63	9	4	14	3
5	99	19	44	4	5	24	9

***Step*1**. The initial discrete values of k_is and m_is are caculated using Eqs. (14)-(15), respectively. Table 2. Contains these values. Go to *Step* 2.

Table 2. Initial values of $\boldsymbol{k_i}$ and $\boldsymbol{m_i}$

Item	1	2	3	4	5
k_i	14	5	14	5	16
m_i	2	5	2	4	2

***Step*2**. As the lower and the upper limit for m_i are set 5 and 35, respectively, the m_i values are obtained in Table 3. As a rsult, the final discrete values of k_is are computed using Eq. (18) as shown in Table 3. Based on the results in Table 3, since $(\sum_{i=1}^{n} f_i m_i k_i = 835) \leq (f = 7900)$, the warehouse space constraint is satidfied. Go to *Step* 7.

Table 3. Final values of $\boldsymbol{k_i}$ and $\boldsymbol{m_i}$

Item	1	2	3	4	5
m_i	5	5	5	5	5
k_i	6	5	7	5	6

***Step*7**. Based on the above solution, the total inventory cost is approximately \$3118.547. The final solution of the problem using the proposed heuristic is shown in Table 4.

Table 4. Final solution of the example used in Pasandideh and Niaki (2008)

Item	k_i	m_i	$Q_i = m_i k_i$	TC
1	6	5	30	
2	5	5	25	
3	7	5	35	\$3118.53703512169
4	5	5	25	
5	6	5	30	

The heuristic algorithm, when coded in Matlab 2015b software, solves the above problem in 0.0013485 seconds, where only one iteration was taken to find the optimal solution. However, the GA utilized in Pasandideh and Niaki (2008) solved this problem in 30 generations, each with evaluating 8 chromosomes. In addition, the total cost that comes from GA used in Pasandideh and Niaki (2008) is approximately $14875.23. This implies that the proposed heuristic algorithm solved the problem in less computational time with a far better quality solution.

5. Conclusion

This paper presented a simple and efficient heuristic algorithm to solve a multi-product EPQ inventory control problem with discrete delivery orders. A simpler and modified model was first developed for the previously published model with less constraints and decision variables. As the model was an NLIP problem hard to be solved analytically, a heuristic algorithm was proposed to solve the problem simpler and faster than the previously published meta-heuristic solution algorithm (GA). In order to show the faster convergence of the proposed heuristic that requires less computational time, the previously published problem was solved using both solution algorithms at the end. We demonstrated that the modified model when solved using the proposed heuristic provides an excellent quality solution in less computational time when compared to the meta-heuristic algorithm. In addition, we showed that the heuristic algorithm can obtain the optimal solution of a single-product production-inventory control problem without constraints.

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